Cosmological term, mass, and space-time symmetry [*]

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In the spherically symmetric case the requirements of regularity of density and pressures and finiteness of the ADM mass m, together with the weak energy condition, define the family of asymptotically flat globally regular solutions to the Einstein minimally coupled equations which includes the class of metrics asymptotically de Sitter as $r \to 0$. A source term connects smoothly de Sitter vacuum in the origin with the Minkowski vacuum at infinity and corresponds to anisotropic vacuum defined macroscopically by the algebraic structure of its stress-energy tensor which is invariant under boosts in the radial direction. Dependently on parameters, geometry describes vacuum nonsingular black holes, and self-gravitating particle-like structures whose ADM mass is related to both de Sitter vacuum trapped in the origin and smooth breaking of space-time symmetry. The geometry with the regular de Sitter center has been applied to estimate geometrical limits on sizes of fundamental particles, and to evaluate the gravito-electroweak unification scale from the measured mass-squared differences for neutrino.

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I. THE EINSTEIN COSMOLOGICAL TERM

Which arrow does fly forever?
An arrow which hits the goal.
Vladimir Nabokov

In 1917 Einstein introduced a cosmological term into his equations describing gravity as space-time geometry (G-field) generated by matter

$$G_{\mu\nu} = -8\pi G T_{\mu\nu} \qquad (1)$$

to make them consistent with Mach's principle, one of his basic motivations [1], which reads: some matter has the property of inertia only because there exists also some other matter in the Universe [2]. When Einstein found that Minkowski geometry is the regular solution to (1) without source term

$$G_{\mu\nu} = 0 \qquad (2)$$

perfectly describing inertial motion in the absence of a matter, he modified his equations by adding the cosmological term $\Lambda g_{\mu\nu}$ in the hope that modified equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \qquad (3)$$

will have reasonable regular solutions only when mattaer is present (if matter is the source of inertia, then in case of its absence there should not be any inertia [3]). The by-product of this hypothesis was the static Einstein cosmology in which the task of Λ was to make a universe

The story of abandoning Λ by Einstein is typically told as dominated by successes of FRW cosmology confirmed by Hubble's discovery of the Universe expansion.

In reality the first reason was de Sitter solution: soon after introducing $\Lambda g_{\mu\nu}$, in the same year 1917, de Sitter found quite reasonable solution to the equation (3) with $T_{\mu\nu} = 0$,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \qquad (4)$$

which made evident that a matter is not necessary to produce the property of inertia [4].

In de Sitter geometry [5]

$$ds^2 = \left(1 - \frac{\Lambda}{3}r^2\right)dt^2 - \frac{dr^2}{1 - \frac{\Lambda}{3}r^2} - r^d\Omega^2 \qquad (5)$$

Λ must be constant by virtue of the contracted Bianchi identities

$$G^{\mu\nu}_{;\nu} = 0 \qquad \to \qquad \Lambda = const \qquad (6)$$

It plays the role of a universal repulsion whose physical sense remained obscure during several decades when de Sitter geometry has been mainly used as a simple testing ground for developing the quantum field technics in a curved space-time.

In 1965 two papers shed some light on the physical nature of the de Sitter geometry. In the first Sakharov suggested that gravitational effects can dominate an equation of state at superhigh densities and that one of possible equations of state for superdense matter is [6]

$$p = -\rho \qquad (7)$$

which formally corresponds to equation of state for $\Lambda g_{\mu\nu}$ shifted to the right-hand side of the Einstein equation (4) as some stress-energy tensor

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \qquad (8)$$

The physical sense of this operation has been clarified in the second paper by Gliner who identified $\Lambda g_{\mu\nu}$ on the basis of the Petrov classification [7], as corresponding to a stress-energy tensor for a vacuum defined by the algebraic structure of its stress-energy tensor [8]

$$T_{\mu\nu} = \rho_{vac} g_{\mu\nu}; \quad \rho_{vac} = (8\pi G)^{-1}\Lambda = const \quad (9)$$

In quantum field theory a vacuum state which behaves like an effective cosmological term

$$< T_{\mu\nu} >=< \rho_{vac} > g_{\mu\nu} \quad (10)$$

was first found by De Witt [9] (for review [10,4]).

Properties of de Sitter geometry ultimately advanced Λ to provide the generic reason for the Universe expansion producing a huge growth of the scale factor [11] sufficient to explain various puzzles of the standard big bang cosmology (for review [12]).

The triumphs of the inflationary paradigm somehow left in shadow the primary sense of Einstein's message of introducing Λ as a quantity which can have something in common with inertia.

The question of possible connection between Λ and inertia, touches the other Einstein's profound proposal - to describe an elementary particle by regular solution of nonlinear field equations as "bunched field" located in the confined region where field tension and energy are particularly high [13].

The possible way to such a structure of gravitational origin whose mass is related to Λ and whose regularity is related to this fact, can be found in the Einstein field equations (1) and in the Petrov classification for $T_{\mu\nu}$. On this way one possible answer comes from model-independent analysis of the minimally coupled spherically symmetric Einstein equations if certain general requirements are satisfied [14–16]:

(a) regularity of density $\rho(r)$,
(b) finiteness of the ADM mass m and
(c1) dominant energy condition for $T_{\mu\nu}$ either
(c2) weak energy condition for $T_{\mu\nu}$ and regularity of pressures.

Cases (c1)-(c2) differ by behavior of the curvature scalar R, which in the first case is non-negative [16].

Conditions (a)-(c) lead to the existence of globally regular geometry asymptotically de Sitter as $r \to 0$, called de Sitter-Schwarzschild geometry in case of Minkowski asymptotic at infinity [17–19] (for review [20]). We applied this geometry to estimate geometrical limits on sizes of fundamental particles in testing to which extent they can be treated as point-like [21], and to study space-time origin of a mass-squared difference and evaluate the gravity-electroweak unification scale from the mass squared differences for neutrino [22,23].

This talk is organized as follows. In Section II we present conditions leading to the existence of geometry with the regular de Sitter center. In Section III we outline the structure of a source term for this geometry and in Section IV geometry itself. Sections V and VI are devoted to tests: estimates of limits on geometrical sizes of leptons and extraction of the gravito-electroweak scale from the data on mass squared differences for neutrino.

II. REGULAR DE SITTER CENTER

We start from the Einstein field equation (1) without cosmological term.

The standard form for a static spherically symmetric line element reads [24]

$$ds^2 = e^{\mu(r)}dt^2 - e^{\nu(r)}dr^2 - r^2 d\Omega^2 \quad (11)$$

where $d\Omega^2$ is the metric on a unit 2-sphere. The metric functions satisfy the Einstein equations

$$8\pi G T_t^t = 8\pi G \rho(r) = e^{-\nu}\left(\frac{\nu'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} \quad (12.1)$$

$$8\pi G T_r^r = -8\pi G p_r(r) = -e^{-\nu}\left(\frac{\mu'}{r} + \frac{1}{r^2}\right) + \frac{1}{r^2} \quad (12.2)$$

$$8\pi G T_\theta^\theta = 8\pi G T_\phi^\phi = -8\pi G p_\perp(r) =$$

$$-e^{-\nu}\left(\frac{\mu''}{2} + \frac{\mu'^2}{4} + \frac{(\mu' - \nu')}{2r} - \frac{\mu'\nu'}{4}\right) \quad (12.3)$$

Here $\rho(r) = T_t^t$ is the energy density (we adopted $c = 1$ for simplicity), $p_r(r) = -T_r^r$ is the radial pressure, and $p_\perp(r) = -T_\theta^\theta = -T_\phi^\phi$ is the tangential pressure for anisotropic perfect fluid [24]. The prime denotes differentiation with respect to r. Integration of Eq.(12.1) gives

$$e^{-\nu(r)} = 1 - \frac{2GM(r)}{r}; \quad M(r) = 4\pi \int_0^r \rho(x)x^2 dx \quad (13)$$

whose asymptotic for large r is $e^{-\nu} = 1 - 2Gm/r$, with the mass parameter m given by

$$m = 4\pi \int_0^\infty \rho(r)r^2 dr \quad (14)$$

The dominant energy condition $T^{00} \geq |T^{ab}|$ for each $a, b = 1, 2, 3$, which holds if and only if [25]

$$\rho \geq 0; \quad -\rho \leq p_k \leq \rho; \quad k = 1, 2, 3 \quad (15)$$

implies that the local energy density is non-negative for any observer in his local frame, and each principal pressure never exceeds the energy density. In the limit $r \to \infty$ the condition of finiteness of the mass (14) requires density profile $\rho(r)$ to vanish at infinity quicker than r^{-3},

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and then, in the case (c1) the dominant energy condition (15) requires both radial and tangential pressures to vanish as $r \to \infty$. Then $\mu' = 0$ and $\mu =$ const at infinity. Rescaling the time coordinate allows one to put the standard boundary condition $\mu \to 0$ as $r \to \infty$ [26] which leads to asymptotic flatness needed to identify (14) as the ADM mass [26]. In the case (c2) we postulate regularity of pressures including vanishing of p_r at infinity sufficient to get $\mu' = 0$ needed for asymptotic flatness.

From Eqs.(12) hydrodynamic equations follow [27,28,14]

$$p_\perp = p_r + \frac{r}{2}p_r' + (\rho + p_r)\frac{GM(r) + 4\pi G r^3 p_r}{2(r - 2GM(r))} \quad (16)$$

$$p_r + \rho = \frac{1}{8\pi G}\frac{e^{-\nu}}{r}(\nu' + \mu') \quad (17)$$

which define, by imposing requirements (a)-(c), asymptotic behavior of a mass function $M(r)$ at approaching the regular center:

$$M(r) \quad \to \frac{4\pi}{3}\rho(0)r^3 \quad as \quad r \to 0 \quad (18)$$

Eq.(17) leads to $\nu' + \mu' = 0$ and $\nu + \mu = \mu(0)$ at $r = 0$ with $\mu(0)$ playing the role of the family parameter [14].

The weak energy condition, $T_{\mu\nu}\xi^\mu\xi^\nu \geq 0$ for any time-like vector ξ^μ, which means non-negative density for each observer on a time-like curve, is contained in the dominant energy condition and satisfied if and only if

$$\rho \geq 0; \rho + p_k \geq 0, k = 1, 2, 3 \quad (19)$$

By Eq.(17) it demands $\mu' + \nu' \geq 0$ everywhere [14]. A function $\mu(r)+\nu(r)$ is growing from $\mu+\nu = \mu(0)$ at $r = 0$ to $\mu + \nu = 0$ at $r \to \infty$, which leads to $\mu(0) \leq 0$ [14]. *

The range of family parameter $\mu(0)$ includes the value $\mu(0) = 0$. In this case the function $\nu(r) + \mu(r)$ is zero at $r = 0$ and at $r \to \infty$, its derivative is non-negative, hence $\nu(r) = -\mu(r)$ everywhere [14], and the metric is

$$ds^2 = g(r)dt^2 - \frac{dr^2}{g(r)} - r^2 d\Omega^2 \quad (20)$$

with

$$g(r) = 1 - \frac{2GM(r)}{r}; \qquad M(r) = 4\pi \int_0^r \rho(x)x^2 dx \quad (21)$$

The metric (20) has Schwarzschild asymptotic as $r \to \infty$

$$ds^2 = \left(1 - \frac{r_g}{r}\right) - \frac{dr^2}{\left(1 - \frac{r_g}{r}\right)} - r^2 d\Omega^2; \quad r_g = 2Gm \quad (22)$$

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*The well known example of solution from this family is boson stars [29] (for review [30]).

The weak energy condition defines also equation of state and thus asymptotic behavior as $r \to 0$ [14]: In the limit $r \to 0$ the equation of state becomes $p = -\rho$, which gives de Sitter asymptotic as $r \to 0$

$$T_{\mu\nu} = \rho_0 g_{\mu\nu}; \quad ds^2 = \left(1 - \frac{\Lambda}{3}r_2\right)dt^2 - \frac{dr^2}{\left(1 - \frac{\Lambda}{3}r_2\right)} - r^2 d\Omega^2 \quad (23)$$

with a cosmological constant

$$\Lambda = 8\pi G\rho_0 \quad (24)$$

where $\rho_0 = \rho(r \to 0)$.

We see that Λ has appeared at the origin although it was not present in the basic equations.

The weak energy condition $p_\perp + \rho \geq 0$ demands monotonic decreasing of a density profile. The simple analysis similar to previous shows that the metric (20)-(21) has not more than two horizons [14].

Summary

Requirements (a)-(c) imposed on the Einstein equations *without cosmological term* lead to the existence of the class of metrics asymptotically de Sitter as $r \to 0$ and asymptotically Schwarzschild as $r \to \infty$, with monotonically decreasing density profile, $\rho' \leq 0$, with the metric function which smoothly evolves between

$$1 - \frac{\Lambda}{3}r^2 \quad \leftarrow \quad g(r) \quad \to \quad 1 - \frac{2Gm}{r}$$

and which has not more than two horizons.

III. STRUCTURE OF A SOURCE TERM

For the class of metrics (20) a source term has the algebraic structure [17]

$$T_t^t = T_r^r; \quad T_\theta^\theta = T_\phi^\phi \quad (25)$$

and the equation of state

$$p_r = -\rho; \quad p_\perp = -\rho - \frac{r}{2}\rho' \quad (26)$$

In the Petrov classification scheme [7] stress-energy tensors are classified on the basis of their algebraic structure. When the elementary divisors of the matrix $T_{\mu\nu} - \lambda g_{\mu\nu}$ are real, the eigenvectors of $T_{\mu\nu}$ are non-isotropic and form a co-moving reference frame with a time-like vector representing a velocity. A co-moving reference frame is thus defined uniquely if and only if none of the space-like eigenvalues λ_a ($a = 1, 2, 3$) coincides with the time-like eigenvalue λ_0. Otherwise there exists an infinite set of co-moving reference frames, which makes impossible to define a velocity in principle. This classification provides

general macroscopic definition of a vacuum by symmetry of its stress-energy tensor.

The stress-energy tensor (9) corresponding to the Einstein cosmological term has the structure [(IIII)] in the Petrov classification (all eigenvalues equal), and is identified as a vacuum tensor due to the absence of a preferred co-moving reference frame [8].

A stress-energy tensor (25) with the structure [(II)(II)], has an infinite set of co-moving reference frames, since it is invariant under rotations in the (r,t) plane. Therefore an observer moving through a medium (25) cannot in principle measure the radial component of his velocity. The stress-energy tensor with the algebraic structure (25) is identified thus as describing a spherically symmetric anisotropic vacuum with variable density and pressures, $T_{\mu\nu}^{vac}$, invariant under boosts in the radial direction [17].

Any source terms for the class of metrics (20) evolves smoothly and monotonically from de Sitter vacuum in the center to Minkowski vacuum at infinity

$$de\ Sitter\quad \Lambda\delta_\nu^\mu\quad \rightarrow\quad 8\pi GT_\nu^\mu\quad \rightarrow\quad 0\quad Minkowski$$

ADM mass m responsible for geometry and identified as a gravitational mass by asymptotic behavior of the metric at infinity, is equal, by the equivalence principle, to inertial mass which is thus related to both de Sitter vacuum trapped inside an object, $\Lambda = 8\pi G\rho_0$, and to reducing of symmetry of a source term (25) from the full Lorentz group in the origin to the radial boosts only. Symmetry of space-time generated by such a source, is

$$r \rightarrow 0\quad de\ Sitter\ group\qquad r \rightarrow \infty\quad Poincare\ group$$

This approach can be extended to the case of de Sitter asymptotic at infinity, with $\lambda < \Lambda$. A density component of a source term is taken as $T_0^0 = \rho(r) + (8\pi G)^{-1}\lambda$, and the metric function $g(r)$ in (20) is then given by [31]

$$g(r) = 1 - \frac{2GM(r)}{r} - \frac{\lambda r^2}{3} \qquad (27)$$

This metric is asymptotically de Sitter at both origin and infinity and has not more than three horizons for the case of two Lambda scales [32]. A source term evolves smoothly and monotonically between two de Sitter vacua with different values of cosmological constant.

$$(\Lambda + \lambda)\delta_\nu^\mu\quad \rightarrow\quad 8\pi GT_\nu^\mu\quad \rightarrow\quad \lambda\delta_\nu^\mu$$
$$De\ Sitter\ vacuum \qquad\qquad de\ Sitter\ vacuum$$

This makes it possible to interpret $T_{\mu\nu}^{vac}$ with the algebraic structure (25) and such asymptotic behavior as corresponding to extension of the algebraic structure of the cosmological term from $\Lambda g_{\mu\nu}$, with Λ=const, to an r−dependent cosmological term

$$\Lambda_{\mu\nu} = 8\pi GT_{\mu\nu}^{vac} \qquad (28)$$

evolving monotonically from $\Lambda_{\mu\nu} = \Lambda g_{\mu\nu}$ at $r = 0$ to $\Lambda_{\mu\nu} = \lambda g_{\mu\nu}$ as $r \rightarrow \infty$ [19].

The advantage of such an extension is that a scalar Λ describing vacuum energy density as $\rho_{vac} = 8\pi G\Lambda$ with ρ_{vac} =const by virtue of the contracted Bianchi identities, becomes explicit related to the appropriate component, Λ_0^0, of an appropriate stress-energy tensor, whose vacuum properties follow from its symmetry, and whose variability follows just from the contracted Bianchi identities which give $\Lambda_{\nu;\mu}^\mu = 0$.

Shifting vacuum tensor $8\pi GT_\nu^\mu$ to the left hand side of Einstein equation (1) we obtain

$$G_\nu^\mu + \Lambda_\nu^\mu = 0 \qquad (29)$$

In Λ_ν^μ geometry a mass m is directly connected to cosmological term $\Lambda_{\mu\nu}$ by the ADM formula (14) which in this case reads

$$m = (2G)^{-1}\int_0^\infty (\Lambda_t^t(r) - \lambda)r^2 dr \qquad (30)$$

where λ is the asymptotic value of Λ_t^t at infinity.

Let us note that this observation does not depend on identification of a vacuum tensor of the algebraic structure (25) as associated with a variable cosmological term (28). Any stress-energy tensor from the considered class generates space-time invariant under de Sitter group in the limit $r \rightarrow 0$. And for any metric from this class the standard formula (14) relates the ADM mass m to both de Sitter vacuum trapped in the origin and breaking of space-time symmetry [14].

IV. DE SITTER-SCHWARZSCHILD GEOMETRY

Here we discuss de Sitter-Schwarzschild geometry (21) with Minkowsi asymptotic at infinity.

The key point of de Sitter-Schwarzschild geometry is the existence of two horizons, a black hole event horizon r_+ and an internal horizon r_-. A critical value of a mass exists, m_{crit}, at which the horizons come together and which puts a lower limit on a black hole mass [18]. De Sitter-Schwarzschild configurations are shown in Fig.1.
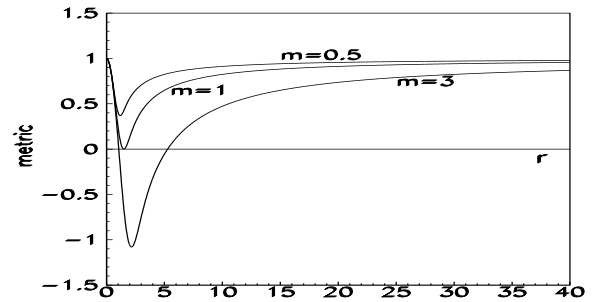

FIG. 1. De Sitter-Schwarzschild configurations. Mass parameter is normalized to m_{crit}.

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For the case of the density profile [17]

$$\rho(r) = \rho_0 e^{-r^3/r_0^2 r_g}; \quad r_0^2 = 3/\Lambda; \quad r_g = 2Gm \quad (31)$$

modelling semiclassically [33] vacuum polarization in the spherically symmetric gravitational field [18]

$$m_{crit} \simeq 0.3 m_{Pl}\sqrt{\rho_{Pl}/\rho_0} \quad (32)$$

For $m \geq m_{crit}$ de Sitter-Schwarzschild geometry describes the vacuum nonsingular black hole [17], whose future and past singularities are replaced with regular cores asymptotically de Sitter as $r \to 0$.

It emits Hawking radiation from both black hole and cosmological horizon with the Gibbons-Hawking temperature $T = \hbar\kappa(2\pi kc)^{-1}$ [34] where κ is the surface gravity. The form of the temperature-mass diagram is generic for de Sitter-Schwarzschild geometry. The temperature on the BH horizon drops to zero as $m \to m_{crit}$, while the Schwarzschild asymptotic requires $T_+ \to 0$ as $m \to \infty$. As a result the temperature-mass curve has a maximum between m_{crit} and $m \to \infty$, where a specific heat is broken and changes sign testifying to a second-order phase transition in the course of Hawking evaporation and suggesting restoration of space-time symmetry to the de Sitter group in the origin [35].

For masses $m < m_{crit}$ de Sitter-Schwarzschild geometry describes a self-gravitating particle-like vacuum structure without horizons, globally regular and globally neutral. It is plotted in Fig. 2 for the density profile (31).
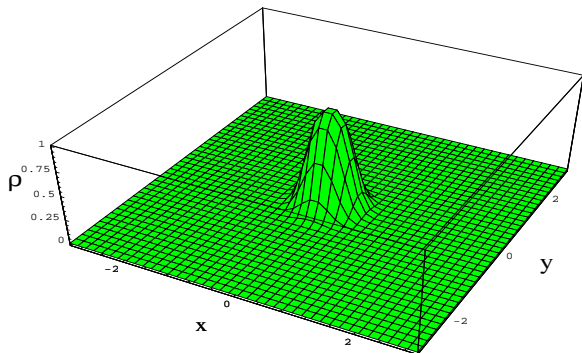

FIG. 2. G-lump in the case $r_g = 0.1 r_0$ ($m \simeq 0.06 m_{crit}$).

It resembles Coleman's lumps - non-singular, non-dissipative solutions of finite energy, holding themselves together by their own self-interaction [36]. G-lump is the regular solution to the Einstein equations, perfectly localized (see Fig. 2) in a region where field tension and energy are particularly high (this is the region of the former singularity). It holds itself together by gravity due to balance between gravitational attraction outside and gravitational repulsion inside of zero-gravity surface $r = r_c$ beyond which the strong energy condition of singularities theorems [25], $(T_{\mu\nu} - Tg_{\mu\nu}/2)\xi^\mu\xi^\nu) \geq 0$, is violated [18]. The surface of zero gravity is depicted in Fig. 3 together

with horizons and with the surface $r = r_s$ of zero scalar curvature $R(r_s) = 0$ which represents the characteristic curvature size in the de Sitter-Schwarzschild geometry in the case (c2).
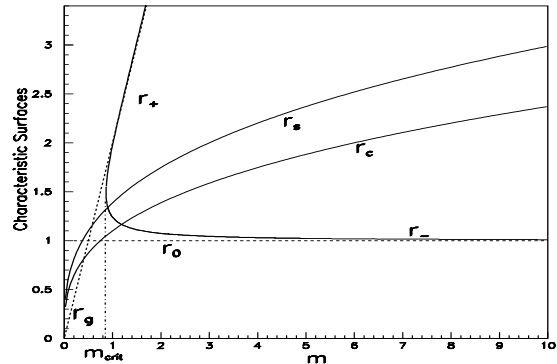

FIG. 3. Horizons of ΛBH, surfaces $r = r_s$ and $r = r_c$.

For the density profile (31) the characteristic size r_s is given by [18]

$$r_s = \left(\frac{4}{3}r_0^2 r_g\right)^{1/3} = \left(\frac{m}{\pi\rho_0}\right)^{1/3} \quad (33)$$

The mass of G-lump is directly connected to a de Sitter vacuum trapped in its center and to breaking of space-time symmetry from the de Sitter group in the origin to the Poincare group at infinity.

This picture conforms with the basic idea of the Higgs mechanism for generation of mass via spontaneous breaking of symmetry of a scalar field vacuum from a false vacuum to a true vacuum state. In both cases de Sitter vacuum is involved and vacuum symmetry is broken.

The difference is that the gravitational potential $g(r)$ (shown in Fig. 4) is generic, and the de Sitter vacuum supplies a particle with mass via smooth breaking of space-time symmetry.
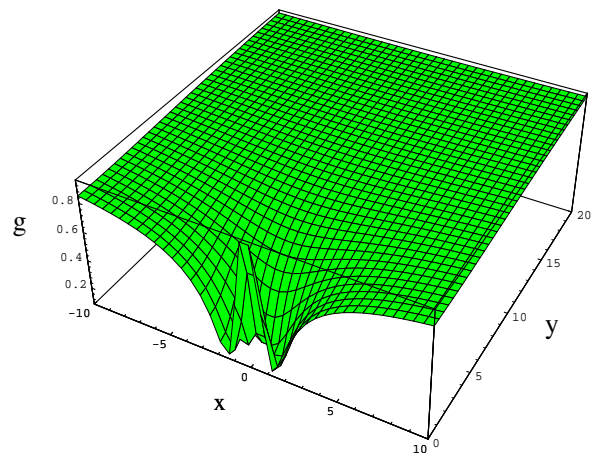

FIG. 4. The gravitational potential $g(r)$ for the case of G-lump with the mass a little bit less than m_{crit}.

Summary

In de Sitter-Schwarzschild geometry de Sitter vacuum is involved generically in mass generation via smooth breaking of space-time symmetry from de Sitter group in the origin to the Poincare group at infinity.

V. TEST 1: LIMITS ON SIZES OF FUNDAMENTAL PARTICLES

To test de Sitter-Schwarzschild geometry we estimate geometrical limits on sizes of fundamental particles by characteristic geometrical size given by curvature radius r_s (see Fig. 3). This implies rather natural assumption that whichever would be particular mechanism involving de Sitter vacuum in mass generation, a fundamental particle may have an internal vacuum core related to its mass and a geometrical size defined by gravity. Geometrical size of an object with the de Sitter vacuum in the origin at the background of the Minkowski vacuum at infinity, can be approximated by de Sitter-Schwarzschild geometry. Characteristic size in this geometry depends on vacuum density at $r = 0$ and actually presents a modification of the Schwarzschild radius r_g to the case when singularity is replaced with the de Sitter vacuum. The resulting difference in sizes is quite impressive: for elementary particles the Schwarzschild radius give sizes which are many orders of magnitude less than l_{Pl}; the characteristic de Sitter-Schwarzschild radius r_s gives sizes close to the experimental upper limits (e.g., $r_s \sim 10^{-18}$ cm for the electron getting its mass from the vacuum at the electroweak scale). In Fig. 5 [21] geometrical sizes r_s plotted by dark triangles, are compared with electromagnetic (EM) and electroweak (EW) experimental limits.
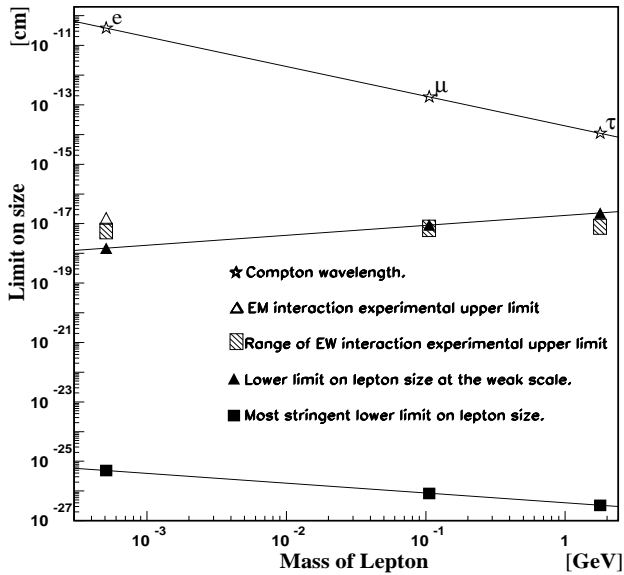

FIG. 5. Characteristic sizes for leptons [21].

In Fig. 5 we show by stars quantum limits given by Compton wave length, by white triangles electromagnetic experimental upper limits coming mainly from reaction $e^+e^- -- \to \gamma\gamma(\gamma)$ [37], by shaded squares experimental electro-weak limits [38], by dark triangles geometrical limits on sizes calculated from Eq.(33) with ρ_0 of the electro-weak scale 246 GeV, and by dark squares the most stringent lower limits on sizes of particles as extended objects. This last limit is calculated by taking into account that in the case when de Sitter vacuum is involved in mass generation, quantum region of a particle localization λ_C must fit within a causally connected region confined by the de Sitter horizon r_0. The scale r_0 is characteristic de Sitter radius related to density ρ_0 by

$$r_0^2 = \frac{3c^2}{8\pi G\rho_0} \tag{34}$$

The requirement $\lambda_C \leq r_0$ gives the limiting scale for a vacuum density ρ_0 related to a given mass m

$$\rho_0 \leq \frac{3}{8\pi}\left(\frac{m}{m_{Pl}}\right)^2 \rho_{Pl} \tag{35}$$

This condition connects a mass of a quantum object m with the scale for vacuum density ρ_0 at which this mass could be generated in principle provided that a mechanism of generation involves de Sitter vacuum.

In Fig. 3 and Fig. 5 theoretical estimates are calculated with using the density profile (31), but the results would not change drastically for different profiles, since a characteristic length scale in any spherical geometry involving de Sitter center, is $r_* \sim (r_0^2 r_g)^{1/3}$ [39].

Let us compare characteristic sizes for an electron, its Compton wavelength, classical and Schwarzschild radius

$$\lambda_C \simeq 3.9 \times 10^{-11}\ cm;\ r_{class} \simeq 2.8 \times 10^{-13}\ cm;$$

$$r_g \simeq 10^{-57}\ cm$$

with lower limits on geometrical sizes for the case when de Sitter vacuum is involved on the electro-weak scale, on gravito-electroweak scale of order of several TeV (see next Section) and at the most stringent scale (35)

$$r_{EW} \simeq 1.5 \times 10^{-18}\ cm;\ r_{GEW} \simeq 2 \times 10^{-23}\ cm;$$

$$r_{lowest} \simeq 5 \times 10^{-26}\ cm$$

We see that numbers given by de Sitter-Schwarzschild geometry are much bigger that the Planck scale $l_{Pl} \sim 10^{-33}$ cm, which justifies application of classical General Relativity for estimation of geometrical sizes of quantum particles.

VI. TEST 2: SPACE-TIME SYMMETRY AS ORIGIN OF MASS-SQUARE DIFFERENCE

In this Section we outline the papers [22,23].

If in the interaction region where particles are created, the interaction vertex is gravito-electroweak, gravity is involved essentially, so geometry around the vertex is not Minkowski any more, and mass is not Casimir invariant of the Poincare group. If density in the vertex is limited, mass of a particle is finite and some of conditions (c) holds, i.e. energy density is non-negative, then geometry around the vertex can be de Sitter. If a false vacuum is somehow involved (for example via Higgs mechanism), then geometry in the interaction region is de Sitter.

If de Sitter group is the space-time symmetry group induced around the gravito-electroweak vertex, then particles participating in the vertex are described by the eigenstates of Casimir operators of the de Sitter group. Their further evolution in Minkowski background requires further symmetry change. We suggest that the "flavor" can emerge due to change in symmetry of space-time between interaction region and propagation region, in particular from de Sitter group around the vertex to the Poincare group outside [22]. This point needs further detailed analysis which is in progress now.

What we can do immediately is to calculate an eigenstate of the first Casimir invariant of the de Sitter group which relates a mass with the vacuum density ρ_0 at the scale of unification.

It reads [40]

$$I_1 = \Pi_\mu \Pi^\mu - \frac{1}{2r_0^2} J_{\mu\nu} J^{\mu\nu} \qquad (36)$$

with

$$\Pi_\mu = \left(1 + \frac{r^2 - c^2 t^2}{4r_0^2}\right) P_\mu + \frac{1}{2r_0^2} x^\nu J_{\mu\nu} \qquad (37)$$

In the interaction region $r^2 - c^2 t^2 \ll r_0^2$ (to be confirmed below), and the operator I_1 is approximated by

$$I_1 \approx P_\mu P^\mu - \frac{1}{r_0^2} \left(\mathbf{J}^2 - bf K^2\right) \qquad (38)$$

where (details in [22]) $J_{ij} = -J_{ji} = \epsilon_{ijk} J_k$ and $J_{i0} = -J_{0i} = -K_i$, with each of the i, j, k taking the values $1, 2, 3$. The $\mathbf{J}$ are then generators of Lorentz rotation and $\mathbf{K}$ are generators of Lorentz boosts.

This gives

$$I_1 \approx P_\mu P^\mu - \frac{\hbar^2}{2r_0^2} \sigma^2 \qquad (38)$$

Its eigenvalues (degenerated) are:

$$I_1' = \mu^2 c^2 - \frac{3\hbar^2}{2r_0^2} \qquad (39)$$

where $\mu^2 c^2$ is the eigenvalue of the first Casimir invariant for the Poincare group.

We see that in the de Sitter geometry the mass eigenvalues depend on the vacuum density ρ_0 responsible for geometry. This allows for negative mass-square for sub-eV particles if gravito-electroweak unification occurs at TeV scales [23]. This also might offer a natural explanation for anomalous results known as "negative mass squared problem" for ν_e [41].

De Sitter symmetry in the gravito-electroweak vertex gives the characteristic mass-square scale

$$\Delta m^2 = \frac{3\hbar^2}{2c^2 r_0^2} \qquad (40)$$

Connecting r_0 with the unification scale M_{unif} through $\rho_0/\rho_{Pl} \sim (M_{unif}/M_{Pl})^4$, we get

$$M_{unif} \sim \left[\frac{1}{4\pi} \left(\frac{\Delta m^2}{M_{Pl}^2}\right)\right]^{1/4} M_{Pl} \qquad (41)$$

which allows us to read off a unification scale from the neutrino mass-square data [22].

The atmospheric neutrino data [42]

$$\delta m_{ATM}^2 = 2.5 \times 10^{-3} eV^2$$

give for the unification scale

$$M_{unif} \sim 16 \ TeV \qquad (42)$$

and mass-squared difference from solar neutrino data [42]

$$\delta m_{ATM}^2 = 2.5 \times 10^{-3} eV^2$$

gives

$$M_{unif} \sim 6.5 \ TeV \qquad (43)$$

These correspond, respectively, to $r_0 \sim 4 \times 10^{-4}$ cm, and $r_0 \sim 2 \times 10^{-3}$ cm. This justifies accuracy of calculations: for a particle with mass $< m >_{\nu_e} = 0.39$ eV, characteristic curvature size is $r_s \sim 10^{-23}$ cm and the Compton wavelength $\lambda_C \sim 10^{-5}$ cm (for curiosity, the Schwarzschild radius is $r_g \sim 10^{-63}$ cm).

VII. DISCUSSION

The main point outlined here is the existence of the class of globally regular solutions to the minimally coupled GR equations with a source term of the algebraic structure (25) interpreted as spherically symmetric anisotropic vacuum with variable density and pressures $T_{\mu\nu}^{vac}$ associated with a time-dependent and spatially inhomogeneous cosmological term $\Lambda_{\mu\nu} = 8\pi G T_{\mu\nu}^{vac}$, whose asymptotic behavior in the origin, dictated by the weak energy condition, is the Einstein cosmological term $\Lambda g_{\mu\nu}$.

7

De Sitter-Schwarzschild geometry describes generic properties of any configuration satisfying (25) and requirements (a)-(c), obligatory for any particular model in the same sense as de Sitter geometry is obligatory for any matter source satisfying (9).

In de Sitter-Schwarzschild geometry space-time symmetry changes smoothly from de Sitter group at the center to the Poincare group at infinity, and the standard formula for the ADM mass m relates it (generically, since a matter source can be any from considered class) to both de Sitter vacuum trapped inside an object and breaking of space-time symmetry.

Applying this geometry to estimating geometrical limits on lepton sizes, we *do not model* a lepton by G-lump, but only use de Sitter-Schwarzschild geometry as a proper instrument to evaluate the geometrical size of an object whose mass has something in common with the de Sitter vacuum in the origin and Minkowski vacuum at infinity.

Application of classical geometry to estimation of sizes of quantum particles is justified by that estimates give numbers which are very orders of magnitude above the Planck scale so that the manifold is perfectly smooth and quantization of gravity is not relevant.

For leptons this is rather rough estimate since more precise geometry is needed which takes into account charge and rotation. We are working on such a geometry.

Applying geometry with the regular de Sitter center to the problem of neutrino mass-square difference we do not apply it literally. The question we address is what is the space-time symmetry group around a gravito-electroweak vertex. In answering this question we base on a view of a vertex as gravito-electroweak, so that gravity is involved essentially and geometry cannot be Minkowski any more. With de Sitter symmetry group in the interaction region, neutrinos are described by eigenstates of the Casimir operators in de Sitter geometry. We find a relation between mass-squared differences for neutrinos and the unification scale for gravity-electroweak vertex, which predicts a TeV scale for unification.

It is clear that the idea of de Sitter group in the interaction region is applicable to particles besides neutrinos [22]. Actually it is simple and general enough to be applicable to *all* particles that participate in interactions where de Sitter vacuum is involved.

VIII. ACKNOWLEDGMENT

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